Seeking Single Top at DØ

Dugan O’Neil

Simon Fraser University

Sambamurti Memorial Lecture (BNL) 17/07/07
Introduction to Particle Physics
Introduction to DØ
Top Quark Physics
Single Top - The Challenge
Decision Trees
Measuring the Cross Section
First Direct Measurement of $|V_{tb}|$
Conclusions
At the smallest distance scales, what is the world made of? How do those components interact?

By convention there is color, By convention sweetness, By convention bitterness, But in reality there are atoms and space.

- Democritus (c. 400 BCE)
An aside: Pie-ology

Pie-ology

At the smallest distance scales, what is a pie made of? How do those components interact to make a tasty dessert?

- Given no list of ingredients nor any description of its preparation, "understand the pie".

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Digitized image from visible-light scatter experiment.
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- Chemically analyze escaping gases. Apply complex neural net (ie. smell)

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- Given no list of ingredients nor any description of its preparation, “understand the pie”.
- Chemically analyze escaping gases. Apply complex neural net (ie. smell)
- I have understood some features of the pie, but I could not make my own...

Digitized image from visible-light scatter experiment.
To understand it, we must eat it.

Pie-ology is a very competitive field.
Elementary Pie-ology

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- Break down the components and analyze each one (with ice cream if possible)

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- Recreate the pie in the laboratory, understand relative abundances of ingredients, describe their interactions.

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- Write GEANT4-based pie-simulator

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- Write GEANT4-based pie-simulator
- Warning: closely-related types of apples are difficult to distinguish. Even the same apple types can taste different if prepared differently.
Of course, particle physicists don’t probe by eating...we use accelerators and detectors to tell us about structure.

Historically we go to ever smaller scales.

On the way down in scale we have discovered hundreds of particles

However, the fundamental ones are few...
Direct searches for what’s not here (secret ingredients)
The Standard Model - List of Ingredients

- Direct searches for what’s not here (secret ingredients)
- Precision measurements of what’s here
The Standard Model - List of Ingredients

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- Precision measurements of what’s here
- Relationships between members of the chart
The Standard Model - List of Ingredients

Direct searches for what’s not here (secret ingredients)

Precision measurements of what’s here

Relationships between members of the chart

New measurements of SM parameters
The Tools

The Tools - Introduction to DØ
The Fermilab Tevatron

- Run II began in March 2001
- Proton-antiproton collisions at 1.96TeV
- Luminosity up to $2.5 \times 10^{32} cm^{-2}s^{-1}$ (so far)
- Integrated Luminosity (recorded) $>2.7 fb^{-1}$ (billions of events recorded)
Tevatron Luminosity

Run II Integrated Luminosity

Apr 2002 – Dec 2006

910 pb⁻¹ analysis
December 2006

370 pb⁻¹ analysis
July 2005

230 pb⁻¹ analysis
Physics Letters B
March 2005

2.13 fb⁻¹

1.79 fb⁻¹

Many thanks to the Accelerator Division
The People
The Physics - Top Quarks
The Tevatron is still the only place to make top quarks. We learn a lot from pair production via the strong interaction.
Single top quark production

- But we have never observed electroweak production!!

**s-channel**

- $\sigma = 0.88 \pm 0.11$ pb
- published limits (95% C.L.):
  - Run II DØ: $< 5.0$ pb
  - Run II CDF: $< 3.1$ pb

**t-channel**

- $\sigma = 1.98 \pm 0.25$ pb
- published limits (95% C.L.):
  - Run II DØ: $< 4.4$ pb
  - Run II CDF: $< 3.2$ pb
Why do we care? - $|V_{tb}|$

- Has never been observed before!
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- Has never been observed before!
- It should happen (if SM is right)
Why do we care? - $|V_{tb}|$

- Has never been observed before!
- It should happen (if SM is right)
- The value of the cross section is a SM test and the **first** measurement of $|V_{tb}|$ - more later
The s-channel and t-channel are sensitive to different new physics.
This looks a lot like single top!

As soon as we discover it, somebody will try to get rid of it....
Why do we care? - Higgs Backgrounds, Top Spin

- Top decays before it can hadronize (no top jets)
- First chance to measure the polarization of a bare quark!

This looks a lot like single top!
As soon as we discover it, somebody will try to get rid of it....
What precisely are we looking for??

- Electroweak production in two main mechanisms at the Tevatron:

**s-channel**

- What to look for (tb):
  - An isolated lepton
  - 2 b-jets
  - Missing transverse energy

**t-channel**

- What to look for (tqb):
  - An isolated lepton
  - 1 or 2 b-jets
  - Missing transverse energy
  - A light-quark jet
So, just find it already!

![Graph showing cross section vs. Higgs mass](image)

- Total inelastic
- mb
- μb
- nb
- pb
- Higgs (ZH + WH)
- single top
- t̅t
- W
- Z
- q bar
- b
- g
- b bar
- q'
- l
- ν

Cross section (barns) vs. Higgs mass (GeV/²)

- $2 \cdot 10^{10}$
- $1 \cdot 10^{7}$
- 6,000
- 600
- ≈1
We HAVE been looking!


plus 7 PhDs.
We have reconstructed data (electrons, muons and jets, etc).
Making the Background Model Agree with Data

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- We have produced a detailed background model.
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- We get upset. We go home and complain to spouse about the injustice of it all.
Making the Background Model Agree with Data

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- Roughly 1 year later....
Agreement is born!

About 2000 of these to look at!
### Percentage of single top \(tb+tqb\) selected events and S:B ratio

(white squares = no plans to analyze)

<table>
<thead>
<tr>
<th>Electron + Muon</th>
<th>1 jet</th>
<th>2 jets</th>
<th>3 jets</th>
<th>4 jets</th>
<th>≥ 5 jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 tags</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>25%</td>
<td>12%</td>
<td>3%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>1 : 3,200</td>
<td>1 : 390</td>
<td>1 : 300</td>
<td>1 : 270</td>
<td>1 : 230</td>
<td></td>
</tr>
<tr>
<td>1 tag</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>21%</td>
<td>11%</td>
<td>3%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>1 : 100</td>
<td>1 : 20</td>
<td>1 : 25</td>
<td>1 : 40</td>
<td>1 : 53</td>
<td></td>
</tr>
<tr>
<td>2 tags</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>2%</td>
<td>1%</td>
<td>0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 : 11</td>
<td>1 : 15</td>
<td>1 : 38</td>
<td></td>
<td>1 : 43</td>
<td></td>
</tr>
</tbody>
</table>
Decision Trees
Improving S/B

- Normally, we look for variables to distinguish signal from background and make a “cut”:
Unfortunately, we have a whole lot of this:
Decision Trees

Train

- Start with all events (first node)
- For each variable, find the splitting value with best separation between children (best cut).
- Select best variable and cut and produce Failed and Passed branches
- Repeat recursively on each node
- Stop when improvement stops or when too few events left. Terminal node = leaf.
Decision Trees

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**Decision Trees**

**Measure and Apply**

- Take trained tree and run on independent fake-data sample, determine purities.
- Apply to Data
- Should see enhanced separation (signal right, background left)
- Could cut on output and measure, or use whole distribution to measure.
Boosting: Why Use a Tree When You Could Use a Forest?
Boosting: Why Use a Tree When You Could Use a Forest?
Decision Trees - Boosting

Boosted Decision Trees

- Decision Tree
- Neural Network
- Random guess
- Cut-Based

Signal efficiency vs. Background efficiency graph.

© R. Schwienhorst
To verify that all of this machinery is working properly we test with many sets of **pseudo-data**.

Wonderful tool to test analysis methods! Run DØ experiment 1000s of times!

Generated ensembles include:

1. 0-signal ensemble \((s + t \sigma = 0pb)\)
2. SM ensemble \((s + t \sigma = 2.9pb)\)
3. “Mystery” ensembles to test analyzers \((s + t \sigma = ??pb)\)
4. Ensembles at measured cross section \((s + t \sigma = \text{measured})\)
5. A high luminosity ensemble

Each analysis tests linearity of “response” to single top.
Decision Trees - Ensembles

\[ \chi^2/\text{ndof} = 4.89/4 \]

\[ \text{Slope} = 1.07 \pm 0.03 \]

\[ \text{Intercept} = -0.12 \pm 0.10 \]
Sensitivity Determination
We use our 0-signal ensemble to determine a significance for each measurement.

**Expected p-value**

In a universe with no single top, how often do we measure at least the SM cross section? (what fraction of 0-signal pseudo-datasets do we measure at least 2.9pb)
We use our 0-signal ensemble to determine a significance for each measurement.

**Expected p-value**

In a universe with no single top, how often do we measure at least the SM cross section? (what fraction of 0-signal pseudo-datasets do we measure at least 2.9pb)

**Observed p-value**

In a universe with no single top, how often do we measure at least the cross section we see in our data.

We also can use the SM ensemble to see how compatible our measured value is with the SM.
Significance/Sensitivity Determination

Decision Trees

SM = 2.9 pb

Probability to rule out background-only hypothesis

Zero-signal ensemble

Pseudo-datasets / 0.4 pb

1.9%, 2.1 sigma

tb+tqb Cross Section [pb]
Looking at Data
Looking at Data: Cross-check samples

- “$W+jets$”: $=2$ jets, $H_T(\text{lepton}, E_T, \text{all jets}) < 175$ GeV
- “$ttbar$”: $=4$ jets, $H_T(\text{lepton}, E_T, \text{all jets}) > 300$ GeV
- Shown: $tb+tqb$ DT output for e+jets

*Good agreement of model with data*
Looking at Data: One Channel

Of course, we have 36 different Decision Trees, let’s look at electron, 2 jet, 1 tag:
What if we just stack them up and zoom in?
What if we just stack them up and zoom in?
Measuring the Cross-Section
Measuring the Cross-Section

DØ Run II *preliminary*

Decision Trees

Measured result

Posterior Probability Density [pb⁻¹]

0.3

0.25

0.2

0.15

0.1

0.05

0

2

4

6

8

10

12

tb+tqb Cross Section [pb]

4.9 ± 1.4 pb
Significance (p-value)

A $3.4\sigma$ excess!!

- **Pseudo-datasets / 0.4 pb**
  - $10^4$
  - $10^3$
  - $10^2$
  - $10^1$
  - $10^0$

- **tb + tqb Cross Section**
  - 4.9 pb

- **Probability to rule out background-only hypothesis**

- **Decision Trees**
  - Zero-signal ensemble
Measuring $|V_{tb}|$

Direct access to $V_{tb}$

- Weak interaction eigenstates are not mass eigenstates
- In SM: top must decay to a $W$ and $d$, $s$ or $b$ quark
  - $V_{td}^2 + V_{ts}^2 + V_{tb}^2 = 1$
  - constraints on $V_{td}$ and $V_{ts}$: $V_{tb} = 0.9991$
- New physics that couples to the top quark:
  - $V_{td}^2 + V_{ts}^2 + V_{tb}^2 + V_{tx}^2 = 1$
  - no constraint on $V_{tb}$
Measuring $|V_{tb}|$

- Given that we now have a measurement of the single top cross section, we can make the first direct measurement of $|V_{tb}|$.
- Use the same infrastructure as cross section measurement but make a measurement of $|V_{tb}|^2$ instead of cross section.
- Caveat: assume SM top quark decays.
- Additional theoretical errors are needed (see hep-ph/0408049)

<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>top mass</td>
<td>13%</td>
<td>8.5%</td>
</tr>
<tr>
<td>scale</td>
<td>5.4%</td>
<td>4.0%</td>
</tr>
<tr>
<td>PDF</td>
<td>4.3%</td>
<td>10.0%</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>1.4%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>
Constrain $|V_{tb}|$ to physical region and integrate:

$$0.68 < |V_{tb}| < 1.00$$
Conclusions

Brand new measurement!
- Top quark physics is a rich area to explore!
- We finally have evidence of EW production!
- Decision Trees helped us sort our apples and better understand the pie.
- $s + t$ cross section: $4.9 \pm 1.4\text{pb}$
- $3.4\sigma$ significance!

$|V_{tb}|$

First direct measurement of $|V_{tb}|$!!

$0.68 < |V_{tb}| < 1$

Shameless SFU Plug
Splitting a node

**Impurity** $i(t)$

- maximum for equal mix of signal and background
- symmetric in $p_{\text{signal}}$ and $p_{\text{background}}$
- minimal for node with either signal only or background only
- strictly concave $\Rightarrow$ reward purer nodes

- Decrease of impurity for split $s$ of node $t$ into children $t_L$ and $t_R$ (goodness of split):
  \[
  \Delta i(s, t) = i(t) - p_L \cdot i(t_L) - p_R \cdot i(t_R)
  \]
- Aim: find split $s^*$ such that:
  \[
  \Delta i(s^*, t) = \max_{s \in \{\text{splits}\}} \Delta i(s, t)
  \]

**Examples**

- Gini
  \[
  Gini = 1 - \sum_{i=s,b} p_i^2 = \frac{2sb}{(s+b)^2}
  \]
- Entropy
  \[
  \text{entropy} = - \sum_{i=s,b} p_i \log p_i
  \]
Decision Trees - Boosting

Boosting
- Recent technique to improve performance of a weak classifier
- Recently used on DTs by GLAST and MiniBooNE
- Basic principal on DT:
  - train a tree $T_k$
  - $T_{k+1} = \text{modify}(T_k)$

AdaBoost algorithm
- Adaptive boosting
- Check which events are misclassified by $T_k$
- Derive tree weight $\alpha_k$
- Increase weight of misclassified events
- Train again to build $T_{k+1}$
- Boosted result of event $i$:
  $$ T(i) = \sum_{n=1}^{N_{\text{tree}}} \alpha_k T_k(i) $$

- Averaging dilutes piecewise nature of DT
- Usually improves performance

Decision Trees - In this analysis

Analysis strategy

- Train 36 separate trees:
  - 3 signals \((s,t,s+t)\)
  - 2 leptons \((e,\mu)\)
  - 3 jet multiplicities (2,3,4 jets)
  - 2 \(b\)-tag multiplicities (1,2 tags)

- For each signal train against the sum of backgrounds

- results shown are 12 \(s+t\) trees
Looking at Data - Event Characteristics $M(W, b)$

$DT < 0.3$

$DT > 0.65$

Excess in high DT output region.
Measuring the Cross-Section - Summary

DØ Run II Preliminary

<table>
<thead>
<tr>
<th>Channel</th>
<th>Cross-Section [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>e / 2jets / 1tag</td>
<td>3.3 ±2.4 -2.0</td>
</tr>
<tr>
<td>e / 2jets / 2tags</td>
<td>4.1 ±5.3 -4.1</td>
</tr>
<tr>
<td>e / 3jets / 1tag</td>
<td>3.3 ±3.6 -2.8</td>
</tr>
<tr>
<td>e / 3jets / 2tags</td>
<td>10.3 ±8.4 -9.8</td>
</tr>
<tr>
<td>e / 4jets / 1tag</td>
<td>6.3 ±7.9 -6.3</td>
</tr>
<tr>
<td>e / 4jets / 2tags</td>
<td>16.1 ±17.1 -16.1</td>
</tr>
<tr>
<td>mu / 2jets / 1tag</td>
<td>5.5 ±3.6 -3.1</td>
</tr>
<tr>
<td>mu / 2jets / 2tags</td>
<td>3.4 ±7.7 -3.4</td>
</tr>
<tr>
<td>mu / 3jets / 1tag</td>
<td>3.5 ±4.3 -3.5</td>
</tr>
<tr>
<td>mu / 3jets / 2tags</td>
<td>0.7 ±10.5 -0.7</td>
</tr>
<tr>
<td>mu / 4jets / 1tag</td>
<td>3.8 ±8.5 -3.8</td>
</tr>
<tr>
<td>mu / 4jets / 2tags</td>
<td>16.9 ±17.9 -16.9</td>
</tr>
</tbody>
</table>

Combined (Decision trees)  4.9 ±1.4 -1.4 [pb]

Z. Sullivan PRD 70, 114012 (2004), m_t = 175 GeV
Consistent with SM?

SM Ensemble

<table>
<thead>
<tr>
<th>tbtqb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>RMS</td>
</tr>
</tbody>
</table>

**e+μ-channel**

Full systematics

201 entries above observed cross section

p-value: 1.1e-01

σ: 1.3

Observed tbtqb cross section [pb]
Systematic uncertainties

- Assigned per background, jet multiplicity, lepton flavour and number of tags
- Uncertainties that affect both normalisation and shapes: jet energy scale and tag rate functions ($b$-tagging parameterisation)
- All uncertainties sampled during limit-setting phase

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top pairs normalization</td>
<td>18%</td>
</tr>
<tr>
<td>$W$+jets &amp; multijets normalization</td>
<td>18–28%</td>
</tr>
<tr>
<td>Integrated luminosity</td>
<td>6%</td>
</tr>
<tr>
<td>Trigger modeling</td>
<td>3–6%</td>
</tr>
<tr>
<td>Lepton ID corrections</td>
<td>2–7%</td>
</tr>
<tr>
<td>Jet modeling</td>
<td>2–7%</td>
</tr>
<tr>
<td>Other small components</td>
<td>Few %</td>
</tr>
<tr>
<td>Jet energy scale</td>
<td>1–20%</td>
</tr>
<tr>
<td>Tag rate functions</td>
<td>2–16%</td>
</tr>
</tbody>
</table>
Measuring the Cross Section

Probability to observe data distribution $D$, expecting $y$:

$$y = \alpha l \sigma + \sum_{s=1}^{N} b_s \equiv a \sigma + \sum_{s=1}^{N} b_s$$

$$P(D|y) \equiv P(D|\sigma, a, b) = \prod_{i=1}^{nbins} P(D_i|y_i)$$

The cross section is obtained

$$Post(\sigma|D) \equiv P(\sigma|D) \propto \int_{a}^{b} \int_{b} P(D|\sigma, a, b) Prior(\sigma),$$

- Bayesian posterior probability density
- Shape and normalization systematics treated as nuisance parameters
- Correlations between uncertainties properly accounted for
- Flat prior in signal cross section
Other Results

\[ \text{DØ Run II } * = \text{preliminary} \]

<table>
<thead>
<tr>
<th>Method</th>
<th>( \sigma (p\bar{p} \rightarrow tb+X, tqb+X) ) [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Trees</td>
<td>4.9 (+1.4) (-1.4) (\text{pb} )</td>
</tr>
<tr>
<td>Matrix Elements*</td>
<td>4.8 (+1.6) (-1.4) (\text{pb} )</td>
</tr>
<tr>
<td>Bayesian NNs*</td>
<td>4.4 (+1.6) (-1.4) (\text{pb} )</td>
</tr>
<tr>
<td>Combination*</td>
<td>4.7 (+1.3) (-1.3) (\text{pb} )</td>
</tr>
</tbody>
</table>

\[ \text{N. Kidočakis, PRD 74, 114012 (2006), } m_{\text{top}} = 175 \text{ GeV} \]
\[ \text{Z. Sullivan, PRD 70, 114012 (2004), } m_{\text{top}} = 175 \text{ GeV} \]
Measuring $|V_{tb}|$

**W-t-b Vertex**

- Most general $Wtb$ coupling ($P_{L,R} = (1 \mp \gamma_5)/2$):

  $$\Gamma_{tbW}^{\mu} = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^{\mu}(f_L^1 P_L + f_R^1 P_R) - \frac{i\sigma^{\mu\nu}}{M_W} (f_L^2 P_L + f_R^2 P_R) \right] u(p_t)$$

- SM: $f_1^L = 1$, $f_1^R = f_2^L = f_2^R = 0$
- Effectively measuring strength of $V-A$ coupling $|V_{tb} f_1^L|$, can be $> 1$
### Decision Trees - 49 variables

#### Object Kinematics
- $p_T(jet1)$
- $p_T(jet2)$
- $p_T(jet3)$
- $p_T(jet4)$
- $p_T(best1)$
- $p_T(notbest1)$
- $p_T(notbest2)$
- $p_T(tag1)$
- $p_T(untag1)$
- $p_T(untag2)$

#### Angular Correlations
- $\Delta R(jet1,jet2)$
- $\cos(best1,lepton)_{besttop}$
- $\cos(best1,notbest1)_{besttop}$
- $\cos(tag1,alljets)_{alljets}$
- $\cos(tag1,lepton)_{btaggedtop}$
- $\cos(jet1,alljets)_{alljets}$
- $\cos(jet1,lepton)_{btaggedtop}$
- $\cos(jet2,alljets)_{alljets}$
- $\cos(jet2,lepton)_{btaggedtop}$
- $\cos(lepton,Q(lepton) \times z)_{besttop}$
- $\cos(lepton,btaggedtopframe)_{btaggedtopCMframe}$
- $\cos(lepton,btaggedtopframe)_{btaggedtopCMframe}$
- $\cos(notbest,alljets)_{alljets}$
- $\cos(notbest,lepton)_{besttop}$
- $\cos(untag1,alljets)_{alljets}$
- $\cos(untag1,lepton)_{btaggedtop}$

#### Event Kinematics
- Aplanarity(alljets, $W$)
- $M(W,best1)$ ("best" top mass)
- $M(W,tag1)$ ("$b$-tagged" top mass)
- $H_T(alljets)$
- $H_T(alljets−best1)$
- $H_T(alljets−tag1)$
- $H_T(alljets,W)$
- $H_T(jet1,jet2)$
- $H_T(jet1,jet2,W)$
- $M(alljets)$
- $M(alljets−best1)$
- $M(alljets−tag1)$
- $M(jet1,jet2)$
- $M(jet1,jet2,W)$
- $M_T(jet1,jet2)$
- $M_T(W)$
- Missing $E_T$
- $p_T(alljets−best1)$
- $p_T(alljets−tag1)$
- $p_T(jet1,jet2)$
- $Q(lepton) \times \eta(untag1)$
- $\sqrt{s}$
- Sphericity(alljets, $W$)

- **Adding variables does not degrade performance**
- **Tested shorter lists, lose some sensitivity**
- **Same list used for all channels**